The force exerted by a fireball

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The force exerted by a fireball was deduced both from the change of the equilibrium position of a pendulum and from the change in the pendulum oscillation period. That measured force was found to be several times larger than the force exerted by the ions accelerated across the double layer that is assumed to surround the fireball. The force enhancement that is expected by ion-neutral collisions in the fireball is evaluated to be too small to explain the measured enhanced force. Gas pressure increase, due to gas heating through electron-neutral collisions, as recently suggested [Stenzel *et al.*, J. Appl. Phys. **109**, 113305 (2011)], is examined as the source for the force enhancement. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4863958]

I. INTRODUCTION

A fireball (a plasma of a ball shape) is often generated near a positively biased electrode in a (usually) low pressure gas.[1–18] The generation of the fireball is believed to be associated with an excitation of a double layer around the fireball, across which part of the discharge voltage drops and in which electrons acquire the energy for ionizing the gas.[1] One would expect the momentum of the particle flow outward of the ball to equal the momentum that the ions acquire while they are accelerated in the double layer. However, recent measurements[15,18] show that the force exerted by the plasma flow is much larger than the force by the impinging ions. We suggested that the enhancement of the force could result from ion-neutral collisions in the fireball,[15] similarly to what we have shown theoretically to occur in cylindrical plasma that expands axially,[19] and demonstrated experimentally in a magnetized plasma.[20–22] Stenzel *et al.* have recently suggested that it is neutral-gas heating that is the source of the enhanced force in their fireball experiment.[18] We present here measurements of the force exerted by the fireball, and we analyze in detail these suggested mechanisms for force enhancement: ion-neutral collisions and gas heating.

In Sec. II, we describe the experimental setup. In Sec. III, we describe the measurements of the force induced by the fireball. We show that the force over the ion flux is several times larger than what is expected according to the maximum momentum that an ion can acquire from the voltage drop. In Sec. IV, we evaluate the enhancement of the force by ion-neutral collisions in the fireball. We calculate the enhancement first when the neutral density is high so that the ion-neutral collision frequency is independent of the ion velocity and second when the collision frequency depends linearly on the ion velocity.[23,24] The enhancement is proportional to the number of collision mean free paths in the first case and to the square root of that number in the second case.[21] The enhancement calculated both ways is too small for explaining the measured force. In Sec. V, we estimate the force by the increased gas pressure by gas heating due to electron-neutral collisions, as suggested in Ref. 18. We employ a model in which the neutral gas is accelerated by the local gas-pressure increase. The rate of gas heating that results in an increase of the gas pressure seems to reasonably explain the measured force as suggested in Ref. 18, if there is a large neutral-gas flow outward of the fireball.

II. EXPERIMENTAL SETUP

A fireball is generated when a DC voltage is applied between an anode and an electron-emitting cathode, both immersed in a low (several mTorr) pressure gas. The anode and cathode in our case were part of our Radial Plasma Source (RPS).[20,21] The RPS was located at the center of a cross ISO 320 vacuum chamber which was pumped to a base pressure of 0.01 mTorr by a two-stage pump station.

The RPS, shown in Fig. 1, consisted of a ceramic insulator, a molybdenum anode, a magnetic-field generating solenoid (not used here), an iron core, a gas distributor, and a cathode. The ceramic insulator was composed of two annular disks connected with an axial segment. The outer diameter of each of the annular disks was 77 mm, the inner diameter was 30 mm, and the axial distance between the two disks was 5 mm. The RPS axial dimension was 30 mm. The molybdenum cylindrical anode was of 48 mm in diameter, 4.5 mm in height, and 0.25 mm in thickness. As is seen in the figure, the ceramic unit, the solenoid, and the iron core have the same axis of symmetry. The empty cylindrical volume between those three parts plays a role of a gas distributor, through which the working gas was supplied through six holes in the ceramic insulator into the space between the two

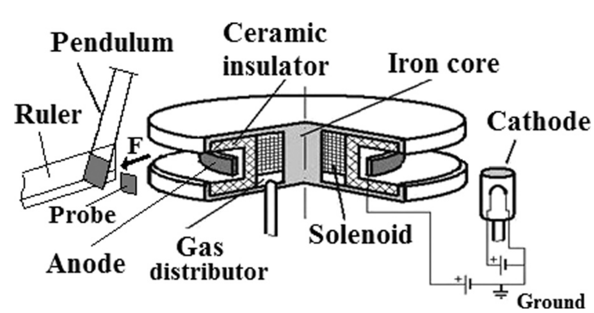

FIG. 1. The RPS, the pendulum, and the Langmuir probe.

annular disks. The gas used was argon with a flow rate in all the experiments of 35 SCCM (Standard Cubic Centimeter per Minute) and, consequently, the pressure near the wall of the vacuum chamber was 5 mTorr. For the plasma generation, we employed a cathode located 80 mm from the axis of the RPS. The electron emission was achieved by heating a tungsten five-turn loop of 6 mm in diameter and 15 mm in height by a DC current of 19 A. An argon gas of 4 SCCM flow rate flows through the cathode. The cathode design and operation were described previously.[20]

The measurement system consisted of a flat Langmuir probe and a pendulum. The flat Langmuir probe was employed for measuring the ion particle flux flowing from the RPS, while the pendulum was used for evaluation of the force exerted by the ion and neutral flow out of the source. Both Langmuir probe and pendulum were located in the vicinity of the fireball.

III. THE ION PARTICLE FLUX AND FORCE EXERTED BY THE FLOW

When the discharge was ignited in an unmagnetized mode at about 5 mTorr, a fireball appeared, as shown in Fig. 2. The size of the ball grew with the discharge current. The discharge current was $I_D = 0.6$ A in Fig. 2(a), 1.3 A in Fig. 2(b), and 1.4 A in Fig. 2(c), and the ball diameter varied from about 24 mm in Fig. 2(a) to about 34 mm in Fig. 2(c). The ball was located for the lower currents as shown in Figs. 2(a) and 2(b), and when the current was increased to above 1.3 A, the ball jumped to the location shown in Fig. 2(c). We will discuss in this paper mostly measurements taken while the ball was in the position shown in Fig. 2(c).

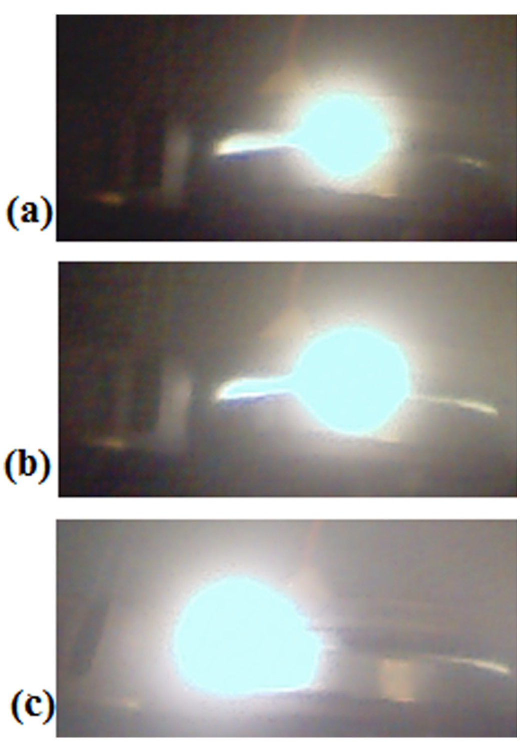

FIG. 2. The fireball near the RPS. (a) and (b) The fireball for a discharge current of 0.6 A, and 1.3 A, respectively. (c) The fireball after moving to a different location for a discharge current of 1.4 A.

A flat Langmuir probe was positioned in front of the fireball shown in Fig. 2(c), at a distance 8 cm from the RPS axis. The ion saturation current from the side of the probe facing the fireball was deduced from measurements as described in Ref. 20. This current was found to be $I_{probe,i} = 1.7$ mA when the ball was in the position in Fig. 2(c). The area of the Langmuir probe was $A_{probe} = 337$ mm^2 and the distance between the Langmuir probe and the ball surface was about 5 mm. The Langmuir probe has been chosen to be of dimensions comparable to those of the pendulum sensing plate, so that the ion current collected by the probe be similar to the ion current impinging on the pendulum plate. We assume that ions flow radially outward of the fireball at about the same flux density all over the ball surface. Employing the ratio of a sphere area of a radius that is the distance of the probe from the ball center (6082 mm^2) and the Langmuir probe area (337 mm^2), we estimate the total ion current out of the fireball to be $I_i = 31$ mA.

The ratio of the electron current I_e (which equals approximately the discharge current, $I_D = 1.4$ A) to the total ion current is $I_e/I_i = 46$. This ratio is smaller than $\sqrt{m_i/m_e} = 271$ (where m_e and m_i are the electron and argon ion masses), as it should be according to the Bohm relation for a double layer.[1,25] However, as was also noted by Stenzel *et al.*,[17] electrons probably cross the double layer at the ball boundary in both directions. The momentum balance across such a layer is then satisfied for a smaller net electron current. The Bohm ratio is not expected to be a valid constraint on the ion current in the fireball.

Employing a pendulum as described in Ref. 20, we deduced the force exerted by the mixed ion-neutral flow out of the fireball. The pendulum and a ruler for measuring the pendulum position are also shown in Fig. 1. We denote by x the deviation of the pendulum from its equilibrium position under gravity only, denoted as $x = 0$ (small angle is assumed). In one method, we evaluated the force by measuring the deviation of the pendulum equilibrium position. The deviation of the pendulum equilibrium position was measured for two cases. In one case, there was no discharge and the pendulum moved under the force exerted by the neutral-gas flow out of the RPS (and under gravity). The deviation of the pendulum position was $x_g = 6 \pm 1$ mm in this case. In a second case, the discharge was on, and the pendulum moved under both the force exerted by the neutral-gas flow out of the RPS and the force exerted by the mixed ion-neutral flow out of the fireball (and under gravity). The deviation of the pendulum position was $x_p = 15 \pm 1.5$ mm in this case, which is shown in Fig. 2(c). Therefore, the force on the pendulum at the equilibrium position in the first case, F_g, and the force on the pendulum at the equilibrium position in the second case, F_p, were $F_{g,p} = Mgx_{g,p}d/l^2$, where M is the pendulum mass, g is the free-fall acceleration, l and d are the distances from the pendulum axis to the center of the sensing plate and to the pendulum center of mass, respectively. These forces were found to be $F_g = 19 \pm 3.5$ μN and $F_p = 46 \pm 5$ μN.

Since when the discharge was off, the gas flow out of the RPS was approximately azimuthally symmetric with respect to the RPS axis of symmetry, we assume that the dependence on x of the force that the gas flow exerts on the pendulum is of the form: $F_g(R+x_g)/(R+x)$, inversely proportional to the distance from the RPS axis, $R+x$. Here, $R=75$ mm is the distance between the RPS axis of symmetry and the position $x=0$. When the discharge was on, a mixed plasma-neutral flow, assumed spherically symmetric with respect to the fireball center, exerted on the pendulum a force of the form: $F_{fb}\left[(a+x_p)/(a+x)\right]^2$. This force by the mixed plasma-neutral flow is inversely proportional to the square of the distance from the center of the fireball, $a+x$, where $a=17$ mm is the radius of the fireball. We positioned the pendulum so that its equilibrium position under gravity only ($x=0$) is near the outer edge of the fireball. The force by the gas flow out of the RPS is assumed not to be modified when the discharge was on. Thus, when the discharge was on, the total force by the flow out of the RPS and the flow out of the fireball was the sum of these two forces. This total force at $x=x_p$ is $F_g(R+x_g)/(R+x_p)+F_{fb}$ and is equal to the measured force, F_p, so that $F_{fb}=F_p-F_g(R+x_g)/(R+x_p)$. We can therefore express the total force at each x by the known parameters F_g, F_p, x_g, and x_p.

To gain confidence in our assumptions about the spatial dependence of the force, we also used another method. In that other method, we measured the period of oscillations of the pendulum under the force by the flow and the gravitational force, when the discharge is on. The oscillations are shown in the movie in Fig. 3 (Multimedia view). The average period when the fireball was close (Fig. 2(c) and also the movie in Fig. 3) was found from Fig. 4 to be $t_p=0.44\pm0.01$ s. The period of oscillations in vacuum was measured as $t_v=0.56\pm0.01$ s (not shown here).

The equation of motion of the pendulum is

$$I\frac{d^2x}{dt^2}+Mgdx=F_gl^2\left(\frac{R+x_g}{R+x}\right)+F_pl^2\left[1-\frac{F_g}{F_p}\left(\frac{R+x_g}{R+x_p}\right)\right]\left(\frac{a+x_p}{a+x}\right)^2. \quad (1)$$

The pendulum oscillates under three forces. The first and the second terms on the right-hand side (RHS) of the equation are contributions to the torques by the force by the gas flow out the RPS and by the force by the mixed ion-neutral flow out of the fireball, respectively, as described above. The second term on the left hand side is the linear contribution of gravity. The moment of inertia of the pendulum is I.

We now write the equation of motion as

$$\frac{d^2x}{d\tau^2}+(2\pi)^2x=(2\pi)^2x_g\left(\frac{R+x_g}{R+x}\right)+(2\pi)^2x_p\left[1-\frac{x_g}{x_p}\left(\frac{R+x_g}{R+x_p}\right)\right]\left(\frac{a+x_p}{a+x}\right)^2, \quad (2)$$

where $\tau\equiv2\pi\sqrt{Mgd/I}t$. When gravity is the only restoring force of the pendulum, the period of oscillations is $\tau=1$, and $t_v=(\sqrt{I/Mgd})2\pi$. We integrate numerically Eq. (2) with the initial conditions $x(\tau=0)=3$ mm and $dx/d\tau(\tau=0)=0$, and find that the turning point is $x=29$ mm at $\tau=0.375$.

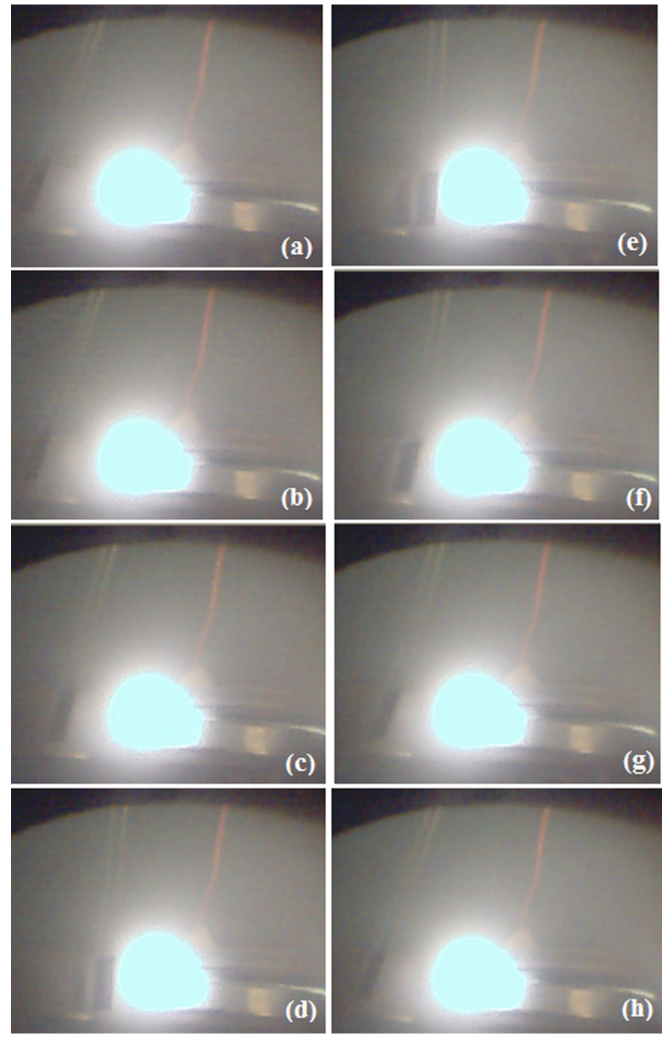

FIG. 3. The plasma and the pendulum interaction (a)-(h) Frames 12 to 19 of the movie that show the fireball and the pendulum at different times within a single oscillation period (Multimedia view). [URL: http://dx.doi.org/10.1063/1.4863958.1]

Therefore, we expect the period of oscillations in the presence of the mixed plasma-gas flow to be $t_p=2\times0.375\times t_v=0.42$ s, while the measured value is $t_p=0.44$ s. This is a reasonable agreement considering the approximations made in the modeling, which indicates that the RHS of Eqs. (1) and (2) describe well the dependence on x of the force by the flow.

Let us estimate the total radial force F exerted on a spherical surface around the fireball, by the flow out of the fireball. We divide F_{fb}, the force on the pendulum by the fireball at $x=x_p$, by the ratio of the area of a sphere of radius $a+x_p$ to the area of the pendulum $A_{pend}=4$ cm^2. We obtain that

$$F=F_p\left[1-\frac{x_g}{x_p}\left(\frac{R+x_g}{R+x_p}\right)\right]\frac{4\pi(a+x_p)^2}{A_{pend}}=9.5\times10^{-4}\,\text{N}. \quad (3)$$

Since we assume that there is no external force on the flow outside the fireball, this is also the expected force on a spherical surface outside the fireball of any radius. In particular,

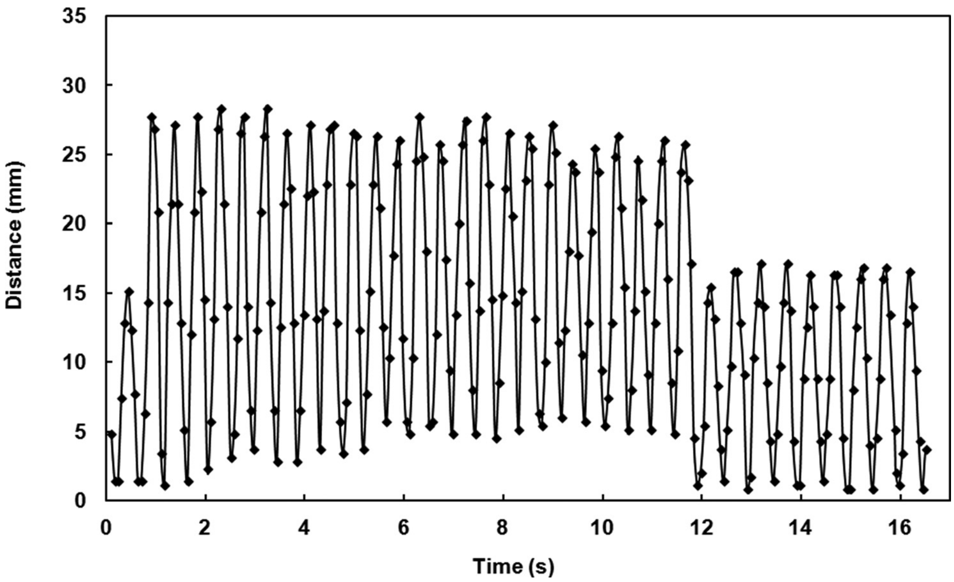

FIG. 4. The pendulum deviation x versus time, as was shown in the movie. The equilibrium position when no force is exerted is denoted as $x = 0$.

this should also be the force on a spherical surface at the location of the Langmuir probe, at $x_{probe} = 5$ mm.

We note here that the anode seems to be heated significantly by the electron current. The hole in the ceramic insulator, through which the gas flows from the gas distributor towards the pendulum, is close to the heated anode. The gas could be heated by the hot anode in addition to heating inside the fireball. To estimate the effect of this additional heating, we measured the force, with no discharge, by the gas flow when it passes along a heated coil of a size and temperature (900 K) similar to those of the anode during the discharge. The force was not enhanced significantly relative to the force by the gas flow when the anode was cold. Therefore, the origin of the force is indeed the momentum gained inside the fireball.

The total ion particle flux Γ_{iT} is determined by the total ion current evaluated above I_i. It is

$$\Gamma_{iT} = \frac{I_i}{e} = 1.9 \times 10^{17}\ \mathrm{s}^{-1}. \tag{4}$$

We now calculate the force per ion, which turns out to be

$$\frac{F}{\Gamma_{iT}} = 5.0 \times 10^{-21}\,\frac{\mathrm{m}}{\mathrm{s}}\mathrm{kg}. \tag{5}$$

We now estimate the momentum of an argon ion that acquired the maximal velocity by acceleration across the voltage drop between the pendulum and the anode. The applied voltage between the cathode and the anode is $V_{app} = 50$ V, while the potential relative to the cathode, measured by our Langmuir probe in the vicinity of the fireball, is 20 V. Therefore, the voltage across the fireball, V, is about 30 V. The ion momentum with the maximal velocity is therefore

$$m_i\sqrt{\frac{2eV}{m_i}} = 8.0 \times 10^{-22}\,\frac{\mathrm{m}}{\mathrm{s}}\mathrm{kg}, \tag{6}$$

where e is the elementary charge. The momentum carried by the flow per ion is

$$\frac{F}{\Gamma_{iT}\sqrt{2eVm_i}} = 6.2 \tag{7}$$

times larger than the force exerted by ions who have acquired a kinetic energy from the full potential drop. We conclude that the exerted force was not only due to the momentum of the ions gained while they cross the voltage drop across the fireball. In Sec. IV, we explore sources for enhancement of the exerted force. One such source for force enhancement is ion-neutral collisions during the acceleration.[20–22] We present two variations of such an enhancement. In Sec. V we discuss the other potential source of the enhanced force which is the increased gas pressure by heating by the electrons, as recently suggested.[18]

IV. ION-NEUTRAL COLLISIONS

A. The plasma dynamics in the ball

The plasma dynamics in the ball is governed by the continuity equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\Gamma_i\right) = S, \tag{8}$$

and by the momentum equations for the ions and electrons

$$0 = neE - m_i\nu\Gamma_i \quad 0 = -enE - \frac{\partial(nT)}{\partial r}. \tag{9}$$

Here, spherical symmetry around the center of the fireball is assumed and r is the distance from that center, Γ_i is the ion particle flux density, and S is the particle source due to ionization. Also, n is the plasma density, T is the electron temperature, ν is the ion-neutral collision frequency, and E is the radial electric field. We add the equations of motion to obtain

$$-\frac{\partial(nT)}{\partial r} = m_i\nu\Gamma_i. \tag{10}$$

In all cases here, we assume isothermal electrons $T = const$. Therefore, from the second of Eq. (9), the potential drop across the quasi-neutral plasma is

$$\Delta V_{pl} = \frac{T}{e}\ln\left(\frac{n_0}{n_s}\right), \tag{11}$$

where n_0 and n_s are the plasma density at the center of the ball and at the sheath boundary, respectively.

B. The linear diffusion equation

We start with the case that the ionization is of the form

$$S = \beta N n, \tag{12}$$

where β is the electron-impact ionization rate constant and N is the (uniform) neutral-gas density.

For simplicity, we first assume that

$$\nu = \sigma N v_T \tag{13}$$

and is constant. Here, σ is the ion-neutral collision cross section and v_T is the neutral-gas thermal velocity. Note that these equations hold even if the electron flow is different from the ion flow so that the current is not zero. We combine Eqs. (8), (10), and (13) to the linear diffusion equation in a spherical symmetry

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right) + k^2 n = 0; \quad k^2 \equiv \frac{\sigma v_T \beta N^2}{c^2}. \tag{14}$$

Here, $c \equiv \sqrt{T/m_i}$ is the ion acoustic velocity. The plasma density, as a solution of Eq. (14), is

$$n = n_0 j_0(kr) = n_0 \frac{\sin(kr)}{kr}, \tag{15}$$

while, from Eqs. (10) and (13), the radial ion particle flux density is

$$\Gamma_i = \frac{c^2 n_0 k}{\sigma v_T N}\left[\frac{\sin(kr)}{k^2r^2} - \frac{\cos(kr)}{kr}\right], \tag{16}$$

where j_0 is the spherical Bessel function of the first kind. The plasma density is small at the edge, at $r = a$, so that we approximate at this diffusion limit

$$ka = \pi \Rightarrow \frac{\sigma v_T \beta N^2}{c^2}a^2 = \pi^2. \tag{17}$$

This relation determines the value of the electron temperature, if all other quantities are specified, as is often the case in weakly ionized plasmas.[24,26–28]

Using the relation (17), we express the plasma density and ion radial flux density as

$$n = n_0 \frac{\sin(\pi r/a)}{(\pi r/a)} \tag{18}$$

and

$$\Gamma_i = \frac{n_0 T \pi}{m_i v_T \sigma N a}\left[\frac{\sin(\pi r/a)}{(\pi r/a)^2} - \frac{\cos(\pi r/a)}{(\pi r/a)}\right]. \tag{19}$$

Note that Γ_i has a maximum somewhere between the ball center and edge.

The ion flux through a radial envelope, $\Gamma_{ir} \equiv 4\pi r^2 \Gamma_i$, is

$$\Gamma_{ir} = \frac{4 n_0 T a}{m_i v_T \sigma N}\left[\sin\left(\frac{\pi r}{a}\right) - \frac{\pi r}{a}\cos\left(\frac{\pi r}{a}\right)\right]. \tag{20}$$

The maximal ion flux is at the edge of the ball

$$\Gamma_{iT} = \Gamma_{ir}(r = a) = 4\pi \frac{n_0 T a}{m_i v_T \sigma N}. \tag{21}$$

We note on passing that we may write, as in standard particle balance analyses,[24,28] $\Gamma_{iT} = n_s c 4\pi a^2$, where n_s, the plasma density at the sheath edge, is here $n_s = n_0 c/(v_T \sigma N a)$. Therefore,

$$\Delta V_{pl} = \frac{T}{e}\ln\left(\frac{v_T \sigma N a}{c}\right). \tag{22}$$

Here, $\sigma N a$ is the number of collision mean-free-paths within the fireball radius.

The force that acts on the ions outward is

$$\begin{aligned} F_r &= -\int_0^r T\frac{\partial n}{\partial r}4\pi r^2 dr \\ &= \left\{\frac{8a^2}{\pi}\left[1 - \cos\left(\frac{\pi r}{a}\right)\right] - 4\pi r a \sin\left(\frac{\pi r}{a}\right)\right\} n_0 T. \end{aligned} \tag{23}$$

The total radial force on the fireball is

$$F_r = \frac{4}{\pi^2}4\pi a^2 n_0 T, \tag{24}$$

which is the plasma pressure multiplied by the area and by the numerical factor $4/\pi^2$.

The force per ion is therefore

$$\frac{F_r}{\Gamma_{iT}} = \frac{4}{\pi^2} m_i v_T \sigma N a \tag{25}$$

and is proportional to the number of mean free paths within the ball radius.

In Subsection IV C, we redo the calculation assuming a uniform ionization.

C. Ion-neutral collisions in a high density gas—uniform ionization

We assume that the ionization is due to a high energy electron beam of a uniform density (that is smaller than the plasma density n). Thus, the rate of ionization does not depend on the plasma electron density but rather it is

$$S = S_0 = \Gamma_e \sigma_{ion} N, \tag{26}$$

where Γ_e is the electron beam flux density, and σ_{ion} is the electron-energy-dependent ionization cross section. We assume that the source S_0 is uniform inside the fireball, since σ_{ion} is uniform there, as all the electrons in the beam have a similar kinetic energy that they acquired while they were accelerated up the double layer that surrounds the fireball. From Eq. (8), it follows that

$$\Gamma_i = \frac{S_0 r}{3}, \tag{27}$$

so that the total ion flux out of the ball becomes

$$\Gamma_{iT} = \Gamma_{ir}(r = a) = \frac{4\pi S_0 a^3}{3}. \tag{28}$$

Assuming a high neutral gas density, so that the ion-neutral collision frequency is given by Eq. (13), and using Eqs. (10) and (27), we obtain that the plasma density is

$$n = n_0\left(1 - \frac{r^2}{a^2}\right); \quad n_0 = \frac{\nu S_0 a^2}{6c^2}, \tag{29}$$

so that

$$\Gamma_{iT} = 8\pi \frac{n_0 T a}{m_i v_T \sigma N}. \tag{30}$$

We comment, as we did above, that we may write $\Gamma_{iT} = n_s c 4\pi a^2$, where n_s, the plasma density at the sheath edge, is in this case $n_s = 2n_0 c/(v_T \sigma N a)$, twice the value of the previous case. The expression for the potential drop across the plasma is

$$\Delta V_{pl} = \frac{T}{e}\ln\left(\frac{v_T \sigma N a}{2c}\right), \tag{31}$$

similar to the expression in the previous case.

We turn to the estimate of the force

$$F_r = -\int_0^a T\frac{\partial n}{\partial r} 4\pi r^2 dr = 0.5 \times 4\pi a^2 n_0 T, \tag{32}$$

which is the plasma pressure multiplied by the area and by the numerical factor 0.5 instead of $4/\pi^2$ in the case above [Eq. (24)]. The force over the ion flux turns out to be

$$\frac{F_r}{\Gamma_{iT}} = \frac{m_i v_T}{4}\sigma N a, \tag{33}$$

which is also similar to that in the previous case [Eq. (25)], aside from a numerical factor. The force over the ion flux (or the impulse per ion) is therefore very similar when two different forms of S are assumed, as long as the ion-neutral collision frequency was of the same form [Eq. (13)].

We turn now to a third case, in which the neutral gas is of a lower density, so that the ion-neutral collision frequency depends on the ion velocity.

D. Ion-neutral collisions in a low density gas - uniform ionization

As in the previous case, we assume a uniform ionization [Eq. (26)] so that the total flux is given by Eq. (27). However, we assume that the neutral-gas density is low enough so that the relative velocity between the colliding ion and neutral is the ion drift velocity v. The ion-neutral collision frequency in this case is[23,24,28]

$$\nu = \sigma N v. \tag{34}$$

The momentum equation, Eq. (10), becomes

$$\frac{c^2}{\sigma N}\frac{\partial n}{\partial r} = -\frac{\Gamma_i^2}{n}, \tag{35}$$

and with the expression in Eq. (27), we write that

$$n = n_0\left(1 - \frac{r^3}{a^3}\right)^{1/2}; \quad n_0 = \sqrt{\frac{2}{3}}\frac{S_0 a}{3c}\sqrt{\sigma N a}. \tag{36}$$

The total ion flux is therefore

$$\Gamma_{iT} = \frac{4\pi S_0 a^3}{3} = 4\pi a^2 \frac{n_0 c}{\sqrt{\sigma N a}}\sqrt{\frac{3}{2}}, \tag{37}$$

and the density at the sheath boundary in this case is $n_s = n_0/\sqrt{2\sigma N a/3}$. The expression for the potential drop across the plasma becomes

$$\Delta V_{pl} = \frac{T}{e}\ln\sqrt{\frac{2\sigma N a}{3}}. \tag{38}$$

We turn to calculating the force

$$F_r = -\int_0^a T\frac{\partial n}{\partial r}4\pi r^2 dr = 0.740 \times 4\pi a^2 n_0 T, \tag{39}$$

where we used $(3/2)\int_0^1 s^4(1 - s^3)^{-1/2} ds = 0.740$. We find that the impulse per ion is

$$\frac{F_r}{\Gamma_{iT}} = 0.604 m_i c\sqrt{\sigma N a}. \tag{40}$$

The impulse per ion is enhanced by the square root of the number of mean free paths.

E. Estimating the force due to ion-neutral collisions

We employ the theoretical modeling of Subsections IV A–IV D to examine the possibility that the force by the fireball is enhanced in our experiment by ion-neutral collisions. We compare the calculated ratio F_r/Γ_{iT} with the measured value. Comparing the values of the ratio is useful, since the calculated ratio is independent of the plasma density, which we have not measured. The neutral-gas pressure was 5 mTorr near the wall of the vacuum chamber. From our previous studies of the force by the RPS flow when the plasma was magnetized,[22] we estimate the pressure in the region between the plates to be up to 25 mTorr, which, for the gas temperature $T_g = 300$ K, corresponds to $N = 8.5 \times 10^{20}\,\mathrm{m^{-3}}$. Since the fireball is close to the RPS but not between the plates, we estimate that the gas pressure is between 5 and 25 mTorr (so that N is between 1.7 and $8.5 \times 10^{20}\mathrm{m^{-3}}$, for $T_g = 300$ K). The cross section for ion-neutral collisions is $\sigma = 8 \times 10^{-19}\,\mathrm{m^2}$, and therefore the number of collision mean-free-paths across the fireball radius, σNa, is larger than unity for $N \geq 7.3 \times 10^{19}\,\mathrm{m}^{-3}$.

We first assume that the model in Secs. IV B and IV C hold and we calculate the neutral-gas density necessary for the impulse per ion to be the measured value $F/\Gamma_{iT} = 5.0 \times 10^{-21}\,\mathrm{m\,s^{-1}kg}$ [Eq. (5)]. The thermal velocity of the argon atoms is $v_T = \sqrt{8T_g/\pi m_i} = 400\,\mathrm{ms}^{-1}$ and therefore the necessary neutral-gas density according to Eq. (25) is

$$N = \frac{\pi^2 F_r/\Gamma_{iT}}{4 m_i v_T \sigma a} = 3.4 \times 10^{22}\,\mathrm{m}^{-3}, \quad (41)$$

about 40 times larger than our estimated upper bound on the neutral density. The necessary neutral-gas density according to Eq. (33) is even higher.

We now use the expression in Eq. (40), assuming that the gas density is low enough so that the model in Sec. IV D holds. We estimate the electron temperature inside the fireball. As written above, the voltage across the fireball is about 30 V. We assume that the electron temperature is $T = 10\,\mathrm{eV}$. The ion acoustic velocity of the argon atoms is then $c = 4880\,\mathrm{m\,s}^{-1}$. The necessary neutral-gas density for the impulse per ion F_r/Γ_{iT} to be the measured value [Eq. (5)] according to Eq. (40) is then

$$N = \frac{1}{\sigma a}\left(\frac{F_r/\Gamma_{iT}}{0.604 m_i c}\right)^2 = 4.8 \times 10^{22}\,\mathrm{m}^{-3}, \quad (42)$$

which is about 55 times larger than our estimated upper bound on the neutral density.

We conclude that ion-neutral collisions and the resulting enhanced plasma pressure contribute only a small part to the force exerted by the flow. We therefore examine the possibility that, as suggested in Ref. 18, it is gas heating inside the fireball by the energetic electrons that increases the gas pressure, and, consequently, the exerted force. For the calculation of the gas heating, we do need to know the plasma density.

We use our measurements to estimate the plasma density in the fireball. From Eq. (37), we can write the maximal plasma density as

$$n_0 = \sqrt{\frac{2}{3}}\frac{\Gamma_{iT}\sqrt{\sigma N a}}{4\pi a^2 c}. \quad (43)$$

For N between 1.7 and $8.5 \times 10^{20}\,\mathrm{m}^{-3}$, n_0 varies from $1.3 \times 10^{16}\,\mathrm{m}^{-3}$ to $3.0 \times 10^{16}\,\mathrm{m}^{-3}$. In the following analysis, we use this estimate of the plasma density.

In Sec. V, we calculate the possible contribution of gas heating to the exerted force.

V. EXERTED FORCE DUE TO GAS HEATING

We evaluate here the enhancement of the exerted force due to gas heating. Stenzel *et al.*[18] evaluated that force for a pulsed fireball. We have found that a force is exerted even for a steady-state fireball. We assume that the force is due to a neutral-gas flow outward of the heated fireball. If a steady-state force is exerted by a particle flow constantly exiting the fireball, there must also be a particle flow of the same flux into the fireball. We estimate here what the power deposited in the fireball and the particle flux into and out of the fireball should be, in order to exert the force that we measured.

The total force exerted by the flow exiting the plasma is

$$F_T = \Gamma_{NT} m_i u_1, \quad (44)$$

where Γ_{NT} is the total neutral-gas flux outward of the fireball, and u_1 is the flow velocity upon exiting the fireball. The power that was deposited in the flow inside the fireball and that is carried by the flow outward is

$$P_T = \Gamma_{NT} m_i \frac{u_1^2}{2}. \quad (45)$$

We assume that the velocity of the neutral-gas atoms entering the fireball is much smaller than u_1. The part of the power carried by the flow outward that equals $(5/2)\Gamma_{NT}T_{g1}$ equals the power carried by the flow entering the fireball and does not contribute to the net power flow (T_{g1} is the gas temperature at the fireball boundary). From the expressions for the force and for the power, we obtain the relation

$$F_T = (2 m_i \Gamma_{NT} P_T)^{1/2}. \quad (46)$$

We can express the flux as $\Gamma_{NT} = N_1 u_1 A$, where A is the area of the ball from which the gas flows outward and N_1 is the density of the neutral-gas density upon exiting the fireball. We assume that the source of the power is the gas heating by the electron current. The collision rate, the number of electron-neutral collisions in the fireball per unit time per unit volume, is $\langle\sigma_{eN} v_{Te}\rangle Nn$, where $\langle\sigma_{eN} v_{Te}\rangle \cong 5 \times 10^{-13}\,\mathrm{m^3\,s^{-1}}$ is the argon electron-neutral collision rate constant (Fig. 3.16 in Ref. 24 for $T \geq 10\,\mathrm{eV}$). The energy deposited in the gas inside the fireball per unit time per unit volume, Q, is the product of the collision rate by E_e, the average electron energy, and by $2\,m_e/m_i$, the fraction of electron energy that is transferred to an argon atom in one collision

$$Q = E_e \frac{2 m_e}{m_i}\langle\sigma_{eN} v_{Te}\rangle Nn. \quad (47)$$

We assume that the mean free path for electron-neutral collision is smaller than the ball radius, so that the electrons do not cross the plasma as a beam. Therefore, for simplicity, we assume here that there is no distinction between beam and plasma electrons, all having an average energy $E_e = (3/2)T = 15\,\mathrm{eV}$. We further approximate the value of N in Eq. (47) as N_1, its value at the fireball boundary and the value of n as n_0 calculated according to Eq. (43). Therefore, the heating Q is approximately uniform inside the fireball.

Because gas has to flow into the fireball across parts of its spherical boundary and outward of the fireball at other parts of the spherical boundary, the flow cannot be spherical symmetric as we assumed above for the source of exerted force. We therefore do not assume here that the gas flow is spherically symmetric. Rather, we simplify the calculation by approximating the fireball as a cylinder through which the gas is accelerated parallel to its axis. We do not know how good the approximation as a cylinder is, but it is enough for our purpose to examine if a force on the order of the measured force can be modeled. The assumed cylinder is of length a and cross section $A=\pi a^2$, of a volume 3/4 of the fireball volume. The power is therefore $P_T = Q\pi a^3$ and is a function of N_1. We thus obtain an equation for the neutral-gas density, N_1, which is

$$N_1 P_T^2 = \frac{F_T^3}{4m_i A}. \quad (48)$$

We also assume that the effective area for calculating the total force is A and not the fireball area, $4\pi a^2$. These assumptions are different from our previous assumption that the plasma flow outward of the spherical fireball is the source of the force. With these new assumptions, the force is $F_T = 9.5\times10^{-4}\,\mathrm{N}/4$, where $9.5\times10^{-4}\,\mathrm{N}$ was the value we assumed for a force at the fireball surface area [Eq. (3)]. We solve Eq. (48) for N_1, calculate the power $P_T(N_1)$, use Eq. (46) to find Γ_{NT}, and consequently u_1. We find that

$$N_1 = 6.5\times10^{20}\,\mathrm{m}^{-3}, \quad P_T = 0.0092\,\mathrm{W}, \quad \Gamma_{NT} = 4.6\times10^{19}\,\mathrm{s}^{-1}, \quad u_1 = 78\,\mathrm{m\,s}^{-1}, \quad (49)$$

and also that the rate of heating for a volume unit is $Q=597\,\mathrm{Wm}^{-3}$, and that the maximal plasma density is $n_0=2.8\times10^{16}\,\mathrm{m}^{-3}$.

The calculated values of neutral-gas density and velocity and of the deposited power are reasonable. However, the calculated particle flux is considerably larger than the gas flow rate into the chamber, which is $1.56\times10^{19}\,\mathrm{s}^{-1}$ (for 35 SCCM). The exertion of a steady force by gas acceleration described here must be accompanied by inducing a steady gas flow of a rate Γ_{NT} [as given in Eq. (49)] into the fireball.

We now describe a particular steady flow that is accompanied by acceleration of a gas heated by the plasma, with the parameters deduced above [Eq. (49)]. For simplicity, a 1D model is used, in which the flow variables only depend on z (the coordinate along the axis of the fireball cylinder). Note that z is not the coordinate along the axis of the RPS. The flow is described by the equations of continuity, momentum, and energy

$$\frac{\partial}{\partial z}(Nu) = S_N, \quad N m_i u^2 + N T_g = N_0 T_{g0}, \quad \frac{\partial}{\partial z}\left[Nu\left(\frac{m_i u^2}{2} + \frac{5}{2}T_g\right)\right] = Q' \equiv Q + S_N\frac{5}{2}T_{g1}. \quad (50)$$

Here, u is the gas fluid velocity, T_{g0} is the maximal gas temperature where $u=0$ (T_{g1} the gas temperature at the boundary, as above), and S_N and Q are the particle and heat sources densities. Gas of temperature T_{g1} flows into the fireball through its lateral boundaries and is accelerated along z. In our 1D picture, this two-dimensional particle flow is modeled by the volume particle source of rate S_N. We assume that S_N and Q are uniform and obtain the following relations:

$$\frac{5}{2}\frac{T_{g0}}{m_i} = \frac{Q'}{m_i S_N} = \frac{Q}{m_i S_N} + \frac{5}{2}\frac{T_{g1}}{m_i} = \frac{u^2}{2} + \frac{5}{2}\frac{T_g}{m_i}. \quad (51)$$

From these relations and the momentum balance, we obtain an equation of state that relates the gas pressure P_g to the gas density

$$P_g \equiv N T_g = \frac{5}{4} N T_{g0} - \frac{N_0 T_{g0}}{4}. \quad (52)$$

The acoustic velocity u_m turns out to be constant

$$u_m^2 = \frac{1}{m_i}\frac{\partial P_g}{\partial N} = \frac{5}{4}\frac{T_{g0}}{m_i} = \frac{Q'}{2m_i S_N}. \quad (53)$$

The gas-flow variables are expressed as functions of u and u_m

$$T_g = T_{g0} - \frac{m_i u^2}{5}; \quad \frac{S_N z}{N_0 u_m} = \frac{u/u_m}{1+(u/u_m)^2}; \quad \frac{N}{N_0} = \frac{1}{(1+u^2/u_m^2)}. \quad (54)$$

Substituting the values in Eq. (49) into Eq. (54) and assuming that $T_{g1}=300\,\mathrm{K}$, we find that $S_N = 3.0\times10^{24}\,\mathrm{s}^{-1}\,\mathrm{m}^{-3}$ and that

$$T_{g0} = 305.8\,\mathrm{K}; \quad u_m = 280\,\frac{\mathrm{m}}{\mathrm{s}}; \quad N_0 = 7.0\times10^{20}\,\mathrm{m}^{-3}. \quad (55)$$

The neutral-gas density and temperature are increased locally only slightly in the fireball, and this pressure increase is sufficient to provide the measured force.

Let us estimate the heat conduction. The heat conduction, $-K\vec{\nabla}T$, through the boundaries of our model cylinder of an area $4\pi a^2$, is estimated as

$$P_{cond} = K(T_{g0} - T_{g1})4\pi a, \; K = \frac{T_g}{m_i \sigma_N v_T}, \quad (56)$$

where K is the coefficient of heat conductivity[29,30] and σ_N is the neutral-neutral collision cross-section. According to Table 9 in Ref. 31, and the derivation from thermal conductivity measurements in Fig. 9 of Ref. 32, $\sigma_N=4\times10^{-19}\,\mathrm{m}^2$ in argon. Using the relations obtained above when heat conduction is neglected, $T_{g0} - T_{g1} = m_i u_1^2/5 = 2P_T/(5\Gamma_{NT})$, we write the ratio of the heat flux through the boundaries to the energy deposited in the kinetic energy of the exiting gas, as

$$\frac{P_{cond}}{P_T} = \frac{\pi^2 v_T a}{5\Gamma_{NT}\sigma_N} \cong 0.5. \quad (57)$$

Therefore, heat conduction is not negligible. We conclude that in fact a somewhat larger neutral-gas density and particle flux are needed to explain the measured force due to neutral-gas heating.

VI. SUMMARY

We investigated experimentally and theoretically the force that is exerted by a fireball generated near the anode in our radial plasma source. For the force measurements, we used a pendulum and measured the modification of its equilibrium position and of its oscillation period under the exerted force. The measurements indicated that the impulse per ion is much larger than expected by ion acceleration across the double layer around the fireball. We examined theoretically two regimes in which the force is enhanced by ion-neutral collisions in the fireball (which increase the plasma pressure). We found that in order for the enhanced force to be caused by ion-neutral collisions, the neutral-gas density has to be much larger than the estimated density. Gas pressure increase by electron-neutral collisions inside the fireball was shown to be a possible source for the enhanced force, if a large neutral-gas flux into the fireball exists. Further experiments and modelling are required in order to clearly identify the source of the force exerted by the fireball.

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